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8 Introduction to Functions

8–1 Equations in Two Variables

Objective: To solve equations in two variables over given domains of the variables.

Vocabulary

Ordered pair A pair of numbers for which the order of the numbers is important.

Solution of an equation in two variables An ordered pair of numbers that makes the equation true.

To solve an equation To find the set of all solutions of the equation.

Symbols (a, b) (The ordered pair a, b.)

CAUTION 1 (x, y) is not the same as (y, x); the order is important.

CAUTION 2 The equation $2x + 1 = 5$ is a *one-variable equation* and has one number, $\{2\}$, for its solution. The equation $2x + y = 6$ is a *two-variable equation* and will have pairs of numbers for its solution. The numbers in a solution pair of an equation in two variables are written in the alphabetical order of the variables.

Example 1 State whether each ordered pair of numbers is a solution of $2x + y = 6$.

a. $(1, 4)$ **b.** $(-1, 8)$ **c.** $(2, -2)$ **d.** $\left(\frac{5}{2}, 1\right)$

Solution Substitute each ordered pair in the equation $2x + y = 6$.

a. $(1, 4)$ is a solution because $2(1) + 4 = 6$.

b. $(-1, 8)$ is a solution because $2(-1) + 8 = 6$.

c. $(2, -2)$ is *not* a solution because $2(2) + (-2) \neq 6$.

d. $\left(\frac{5}{2}, 1\right)$ is a solution because $2\left(\frac{5}{2}\right) + 1 = 6$.

State whether each ordered pair is a solution of the given equation.

1. $x - y = 5$
$(6, 1), (3, -2)$

2. $2x + y = 8$
$(3, -2), (-3, -2)$

3. $x + 3y = 6$
$(3, 1)(-3, 3)$

4. $12 - y = 2x$
$(3, 6), (4, 4)$

5. $5x - 3y = 0$
$(3, 5), (-3, -5)$

6. $2x - 4y = 0$
$(2, 1), \left(1, \frac{1}{2}\right)$

7. $3a - 4b = 12$
$(4, 0), (0, 3)$

8. $2m - 3n = 6$
$(6, 2), (9, 4)$

9. $2x + 5y = 18$
$(4, 2), \left(\frac{3}{2}, 3\right)$

10. $5m - 4n = 11$
$(3, 1), \left(2, \frac{1}{4}\right)$

11. $xy = 8$
$\left(16, \frac{1}{2}\right), (-4, -2)$

12. $2xy = 4$
$\left(\frac{1}{4}, 8\right), (-2, -1)$

13. $x^2 + y^2 = 5$
$(2, -1), (3, -2)$

14. $x^2 - y^2 = 10$
$(3, -1), (1, -3)$

15. $x^2 - 2y^2 = 15$
$(5, 5), (4, 1)$

16. $2x^2 + 3y^2 = 30$
$(3, 2), (-3, 2)$

1. yes, yes
2. no, no
3–6. yes, yes
7. yes, no
8–9. yes, yes
10. yes, no
11–12. yes, yes
13. yes, no
14–15. no, no
16. yes, yes

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8–1 Equations in Two Variables *(continued)*

Example 2 Solve $2x + 3y = 6$ for y in terms of x.

Solution

$2x + 3y = 6$

$3y = 6 - 2x$ Subtract $2x$ from both sides of the equation.

$y = \frac{6 - 2x}{3}$ Divide both sides of the equation by 3.

Solve each equation for y in terms of x. Answers may vary.

17. $3x + y = 6$ **$y = 6 - 3x$**

18. $2x - y = 5$ **$y = 2x - 5$**

19. $3x + 2y = 7$ **$y = \frac{7 - 3x}{2}$**

20. $x + 3y = 9$ **$y = \frac{9 - x}{3}$**

21. $4x + 2y = 0$ **$y = -2x$**

22. $5x + 4y = 10$ **$y = \frac{10 - 5x}{4}$**

Example 3 Solve $xy + x = 4$ if x and y are whole numbers.

Solution

1. Solve the equation for y in terms of x.

$$y = \frac{4 - x}{x}$$

2. Replace x with successive whole numbers and find the corresponding values of y. If y is a whole number, you have found a solution pair.

The solutions are $(1, 3)$, $(2, 1)$, and $(4, 0)$.

x	$y = \frac{4 - x}{x}$	Solution
0	denominator = 0	No
1	$\frac{4 - 1}{1} = 3$	$(1, 3)$
2	$\frac{4 - 2}{2} = 1$	$(2, 1)$
3	$\frac{4 - 3}{3} = \frac{1}{3}$	No
4	$\frac{4 - 4}{4} = 0$	$(4, 0)$
Values of x greater than 4 give negative values of y.		

23. $(0, 4), (2, 0), (1, 2)$ **24.** $(0, 7), (2, 1), (1, 4)$
25. $(0, 2), (3, 1), (6, 0)$ **26.** $(5, 0), (3, 1), (1, 2)$

Solve each equation if x and y are whole numbers.

23. $2x + y = 4$

24. $3x + y = 7$

25. $x + 3y = 6$

26. $x + 2y = 5$

27. $2x + 3y = 8$

28. $3x + y = 9$

29. $2x + 3y = 6$

30. $xy = 3$

31. $xy + 1 = 7$

32. $xy + 2 = 9$

33. $xy + y = 3$

34. $xy - 2y = 4$

27. (1, 2), (4, 0) **28.** (3, 0), (0, 9), (2, 3), (1, 6) **29.** (0, 2), (3, 0) **30.** (1, 3), (3, 1)
31. (1, 6), (2, 3), (3, 2), (6, 1) **32.** (1, 7), (7, 1)
33. (0, 3), (2, 1) **34.** (3, 4), (4, 2), (6, 1)

Mixed Review Exercises

Write each number in scientific notation.

1. 28,000,000 **2.8×10^7**

2. 0.00461 **4.61×10^{-3}**

3. 104 million **1.04×10^8**

4. 0.0000325 **3.25×10^{-5}**

5. 37,000 **3.7×10^4**

6. 6,302,000 **6.302×10^6**

Simplify. Give answers in terms of positive exponents.

7. $\frac{4n^2}{2n}$ **$2n$**

8. $(2x)^{-3}$ **$\frac{1}{8x^3}$**

9. $\frac{42x^3y^2}{14x^2y}$ **$3xy$**

10. $\frac{a^{-5}}{a^2}$ **$\frac{1}{a^7}$**